Deconvolution and Analysis of Wide-Angle Longwave Radiation Data From Nimbus 6 Earth Radiation Budget Experiment for the First Year

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#### SUMMARY

One year of longwave radiation data from July 1975 through June 1976 from the Nimbus 6 satellite Earth radiation budget experiment is analyzed by representing the longwave radiation field by a spherical harmonic expansion. The data are from the wide-field-of-view instrument. Results show that the limit of the spherical harmonic representation is 12th degree, based on degree variance plots from 12 months. Degree variance plots also show that most of the power is in the lower degree terms. The axisymmetric (zonal) terms dominate, with their coefficients representing approximately 80 percent of the degree variance. Contour maps of the radiation field for 12 months show the geographical distribution of Earth emitted radiant exitance (W-m<sup>-2</sup>) and reveal areas of high and low emitted radiation. The analysis also shows differences in the Earth emitted radiation between the Northern and Southern Hemispheres, presumably due to land and ocean distributional differences.

### INTRODUCTION

The nonuniform distribution of radiative heating and cooling over the Earth drives the motion of the atmosphere and oceans, and an understanding of the interaction between the radiation and the dynamics of the atmosphere and oceans is required for understanding climate processes. For this reason, several Earth radiation instruments of increasing sophistication have flown on spacecraft. As early as 1960, instruments for measuring the Earth's radiation budget were flown on Explorer 7 (Vonder Haar (1968)). Since then, a number of satellites have made Earth radiation budget measurements. The ESSA meteorological satellites, the Nimbus satellites, NOAA 1, and ITOS 1 are some of the satellites which have made Earth radiation budget measurements. flown on these satellites have provided a wealth of Earth radiation budget data, including longwave measurements, shortwave measurements, wide-field-ofview (WFOV) measurements, and scanning radiometer measurements. Descriptions of these satellites, and analysis of data from them, have been reported by numerous authors such as Raschke et al. (1973), Weaver and House (1979), Ellis et al. (1978), and Green and Smith (1978).

The most recent instrument flown on a satellite for making Earth radiation measurements was the Earth radiation budget (ERB) instrument aboard the Nimbus 6 and Nimbus 7 spacecraft (Smith et al. (1977); Jacobowitz et al. (1979)). The ERB instrument includes two Earth viewing WFOV radiometers, one which measures total irradiance (0.2 to 50  $\mu\text{m}$ ) and another which measures shortwave (SW) irradiance (0.2 to 3.8  $\mu\text{m}$ ). The difference between the two irradiances is the longwave (LW) or Earth emitted radiation. The ERB instrument has been providing WFOV measurements since July 1975. These WFOV data are suitable for studying large-scale processes over large spatial scales. WFOV sensors integrate radiant exitance (W-m^-2) from all portions of the Earth atmosphere systems within their fields of view. For the Nimbus 6 spacecraft at an altitude of about 1100 km, the field of view is the entire disk of the Earth.

A deconvolution technique (Smith and Green (1976); Green and Smith (1978)) is applied in this paper to enhance the resolution of the WFOV measurements. Deconvolution represents the radiant exitance at the top of the atmosphere (denoted by TOA and defined herein to be at an altitude of 30 km) by a truncated series of spherical harmonics. Much of the earlier work on radiation budget from satellites has used the geometric shape factor technique to give radiant exitance at TOA. The two techniques are equivalent for global average estimates, but deconvolution gives more information at higher orders.

The Nimbus 6 ERB WFOV data have been analyzed by various investigators. Jacobowitz et al. (1979) recently analyzed and published the results of the first 18 months of ERB measurements. The inverse square approximation (geometric shape factor) was used to determine the radiant exitance at TOA. Some of the earlier investigations have concentrated on the data for August 1975. One such analysis, reported by Smith et al. (1977), defined the monthly mean radiation field by a global contour plot and the global mean. The data analysis technique also employed the inverse square approximation. Green and Smith (1978) analyzed the same data using the technique of parameter estimation and the concept of deconvolution. The parameter estimation technique used in the analysis by Green and Smith is based on solving a system of simultaneous measurement equations using least squares.

In the present paper, a deconvolution (resolution enhancement) technique is presented to study the annual cycle of the distribution of the Earth's LW radiation. The study is based on 1 year of WFOV data from July 1975 through June 1976. The resolution enhancement technique was used to analyze the data and extract mean radiation fields on a monthly basis. However, instead of considering each measurement individually, the data were averaged over 50 by 50 grid areas at satellite altitude. This averaging technique reduced the computational burden at the expense of smoothing the data. This data set was analyzed to produce radiant exitances at TOA. The deconvolution technique takes advantage of the fact that spherical harmonics are the eigenfunctions of the measurement operator and reduces the radiant exitance field from satelite altitude to TOA by dividing by the appropriate eigenvalues.

### SYMBOLS

- $a_n^m$ ,  $b_n^m$  complex coefficients of spherical harmonics for degree n and order m,  $W-m^{-2}$
- C<sub>m</sub> integral defined by equation (14)
- $C_n^m$ ,  $S_n^m$  real coefficients of spherical harmonics at top of atmosphere for degree n and order m, W-m<sup>-2</sup>
- $\hat{C}_n^m$ ,  $\hat{S}_n^m$  real coefficients of spherical harmonics at satellite altitude for degree n and order m, W-m<sup>-2</sup>
- do differential surface element

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g(\alpha)
           angular response function
h
           satellite altitude above re, 1070 km
I_n^m
           integral defined by equation (16)
           total number of global 50 by 50 grids, 1654
K
k
           grid region
k t
           reflection about equator of kth grid region
           radiance, W-m<sup>-2</sup>-sr<sup>-1</sup>
L
¥
           linear integral measurement operator defined in equation (4)
M
           radiant exitance at top of atmosphere, W-m<sup>-2</sup>
           measurement, W-m-2
m
           average of measurements in kth grid region, W-m<sup>-2</sup>
m_k
N
           degree of spherical harmonic expansion
N_{n}^{m}
           normalizing factor for spherical harmonics of degree n
             order
P_{p}^{m}
           associated Legendre polynomials of degree n and order
R(0)
           limb-darkening function for emitted radiation
           radius to top of Earth atmosphere system, 6408.165 km
re
           integral defined by equation (15)
S_{m}
           time
t
\mathbf{y}_{n}^{m}
           complex spherical harmonics of degree n and order
           real spherical harmonics of degree n and order m
             eqs. (9) and (10))
           cone angle at satellite from nadir to surface element at top of
α
             atmosphere, deg
           Earth central angle between satellite nadir to a point on surface
Υ
             at top of atmosphere, deg
\delta_{\mathbf{n}}^{\mathbf{m}}
           Kronecker delta
           colatitude, deg
Θ
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 $\Theta_{\mathrm{S}}$  colatitude of subsatellite point, deg  $\theta$  zenith angle of exiting ray, deg  $\lambda_{\mathrm{n}}$  nth eigenvalue of measurement operator  $\sigma_{\mathrm{n}}^2$  degree variance for degree n,  $w^2-m^{-4}$   $\Phi$  longitude, deg  $\Phi_{\mathrm{S}}$  longitude of subsatellite point, deg  $\Phi$  azimuth angle of exiting ray, deg  $\Omega$  solid angle at satellite subtended by surface element, sr

#### DATA SAMPLE

The Earth radiation budget (ERB) instrument aboard the Nimbus 6 satellite obtained Earth radiation measurements with wide-angle and scanning narrow-angle radiometers. A description of the ERB instrument is given by Smith et al. (1977). Only measurements with the wide-angle radiometers are considered in this paper. The wide-angle radiometer sensed radiation in two channels; one measured total irradiance (0.2 to 50 µm), the other, shortwaye irradiance (0.2 to 3.8  $\mu$ m). Twelve months of these measurements, beginning July 2, 1975, and ending June 29, 1976, were supplied on magnetic tapes by the National Oceanic and Atmospheric Administration (NOAA). Only the longwave radiation component is analyzed herein, and this component was obtained by subtracting the measured shortwave component from the measured value of total irradiance. All measurements from the data tapes are at satellite altitude; and in the analysis which follows, all measurements are increased by 11 percent as a calibration correction. This correction is discussed by Smith et al. (1977) and accounts for a discrepancy between the ERB experiment fixed wide-angle measurements and the integration of scanning narrow-angle measurements.

For most of the data period (July 2, 1975, to June 29, 1976) the ERB instrument operated on a nominal duty cycle of 2 days on and 2 days off; however, because of operational considerations, this schedule was not rigorously followed. Measurements were taken at 4-second intervals during the time the instrument was operating. However, the data tapes supplied by NOAA and on which the analysis in this paper is based reduced the amount of data by averaging every four measurements. This caused some smoothing and gave irradiance measurements at 16-second intervals. Figure 1 is a 12-month calendar showing the approximate time periods covered by the data set as received from NOAA. Each day on the calendar begins at 12 midnight, and the dark bands represent the approximate time periods during which data were recorded.

### DATA EDITING

Some measurements were edited from the data set because near sunrise and sunset the WFOV sensor was exposed to the Sun (Jacobowitz et al. (1979)). Exposure to the Sun was due to the design of the wide-angle radiometer. In order for the wide-angle radiometer to view the entire Earth disk, the actual field of view is about 8° larger than the Earth disk and includes a thin annulus of deep space. When the satellite is near sunrise or sunset, the Sun comes into view and contaminates the measurements. When the Sun is on the horizon at sunrise and sunset, the Sun zenith angle at the subsatellite point is 121.5°. This is shown in figure 2. Allowing for a 2° margin on each side of the thin annulus resulted in a range of Sun zenith angles from 111.5° to 123.5°. Based on plots of SW and LW measurements near sunrise and sunset for a few orbits during the month of August 1975, deleting measurements when the Sun zenith angle ranges from 111.5° to 123.5° is sufficient for eliminating Sun contaminated measurements.

The Sun contaminated measurements introduce an additional sampling bias in the Nimbus 6 data. (A bias in the data already exists because the orbit of Nimbus 6 is Sun synchronous, meaning that the radiant exitance at any latitude is based on measurements made only at two local times.) Since the Sun contaminated measurements occur at night (Sun zenith  $> 90^{\circ}$ ), the editing out of these data reduces the number of samples at night while leaving the daytime data set intact. If the day and night measurements are significantly different, then the radiant exitance for the zone in question will be biased toward the daytime values. No attempt was made to adjust for this effect.

The zone showing the deficiency in number of measurements depends on the time of year. Figure 3 shows the number of measurements for different zones for the month of September. The maximum deficiency occurs between  $\pm (60^{\circ})$  to 65°) latitude. Figure 3 also shows that the least number of measurements occurs in the  $\pm (80^{\circ}$  to  $85^{\circ})$  zone. The measurements in this zone were all located near ±80° since the orbit only goes to about ±80.1° latitude. The largest number of measurements occurs in the  $\pm (75^{\circ})$  to  $80^{\circ}$ ) zone, and the zones in between are sampled about equally except for the deficiency due to Sun contaminated measurements. The number of zonal measurements is dictated by the inclination of the Sun synchronous orbit of Nimbus 6. The dashed line in figure 3 shows how the number of measurements would change if Sun contaminated measurements were not deleted. Between 6 and 7 percent of the measurements for September were deleted because they were contaminated by the Sun. also be noted from figure 3 that when Sun contaminated measurements are not deleted, the number of measurements at about 650 north latitude is less than at 650 south latitude. This data dropout is due to the data acquisition station in Alaska.

A possible source of error in the longwave measurements which could vary from 1 to  $4 \text{ W-m}^{-2}$  was caused by a thermal transient in the shortwave measurements due to irradiation of the sensor at spacecraft sunset (Green and Smith (1978)). No attempt was made to correct for this transient in the data.

Measurements were also omitted if they were greater than 240 W-m<sup>-2</sup> or less than 50 W-m<sup>-2</sup> at satellite altitude. The criterion for keeping measurements between the range 50 and 240 W-m<sup>-2</sup> was based upon visual inspection of plots showing radiant exitance versus time for all measurements. Another criterion deleted a measurement if its value changed by more than 10 W-m<sup>-2</sup> from the preceding measurement over a 16-second interval. This criterion is based on the decision to omit measurements with values greater than the 3 $\sigma$  value of the difference of all adjacent measurements, where adjacent measurements are not separated in time by more than 16 seconds. The 3 $\sigma$  value for the 3-month period of July, August, and September is 8 W-m<sup>-2</sup>.

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Parts of the 12-month data set had anomalous measurements even after the preceding editing had been completed. The anomalies were discovered by visually inspecting radiant exitance-time plots of all measurements for each month of the 12-month data period. Figure 4 shows a composite of one of these plots which shows some anomalies in the data. The anomalies are characterized by periods of unusually low, unusually high, and erratic radiant exitance values.

Measurements for a 2-hour period on July 10 and a 2.5-hour period on July 17 were deleted because of erratic spiked data. About 2 days of measurements were deleted between July 21 and July 23 because average values suddenly dropped to about 90 W-m<sup>-2</sup>. The reason for this sudden decrease in radiant exitance is not known. Another questionable set of data was omitted on August 16, 1975. Starting at 1000 hours GMT, the measurements gradually decreased in value during a full satellite revolution and then stabilized at values approximately 30 W-m<sup>-2</sup> lower than expected. This problem occurred at the end of a data acquisition period; however, the measurements from the next data period did not exhibit the problem. The reason for the gradual decrease in radiant exitance is not known. Approximately 15 hours of measurements were deleted between September 5 and September 6 because of a sudden increase in radiant exitance. Measurements were deleted from October 23 through October 31 because of operational problems. The net effect was incorrect radiant exitance averages for some grid regions. About 3 hours of data in January 1976 were edited out because of anomalous measurements on January 12. Sixteen hours of measurements on February 19 and 30 hours of measurements between February 28 and February 29 were edited out. Measurements taken during the first half of March were edited The February and March measurements were deleted because of operational problems. Ten hours of measurements on June 14 were edited out because of erratic values of radiant exitance.

After anomalous measurements were deleted from the data, a few questionable points still remained in the data. The following procedure was used in an effort to delete as many of these anomalous measurements as possible. Measurements were grouped into 5° latitudinal bands. Any measurement within a band that differed by more than 20 from the average radiant exitance in that band was edited out. This editing procedure caused about 4 percent of the measurements to be deleted.

The total number of measurements for the 12-month period, on which the analysis in this paper is based, was about 588 000 after all editing. This

figure represents about 30 percent of the maximum number of measurements for the year had the ERB instrument made measurements at 16-second intervals uninterrupted.

### ANALYSIS TECHNIQUE

The ERB wide-angle data were used to estimate the longwave radiative field at the top of the atmosphere (defined herein as 30 km). The radiative field was represented by a spherical harmonic expansion, and the coefficients were estimated with measurement data.

#### Measurement Model

The top of the Earth atmosphere system was approximated by a sphere of radius  $r_e.$  The emitted radiance L leaving any point E on this surface is, in general, a function of colatitude  $\Theta_{\text{r}}$  longitude  $\Phi_{\text{r}}$  zenith angle  $\theta_{\text{r}}$  azimuth angle  $\varphi_{\text{r}}$  and time t. (See fig. 5.) Thus, the measurement m at satellite altitude h, colatitude  $\Theta_{\text{s}}$ , and longitude  $\Phi_{\text{s}}$  is given by

$$m(\Theta_S, \Phi_S, t) = \int_{FOV} L(\Theta, \Phi, \theta, \phi, t) g(\alpha) d\alpha$$
 (1)

where  $\Omega$  is the differential of solid angle at the satellite subtended by the surface element,  $\alpha$  is the cone angle at the satellite from the local vertical to the surface element, and the integration is over the field of view (FOV) of the sensor (Green and Smith (1978)). The function  $g(\alpha)$  is the angular response of the sensor to incoming radiation and, for the ERB fixed wide-angle radiometer, has been modeled as a perfectly black flat-plate sensor, or

$$g(\alpha) = \cos \alpha$$
 (2)

Also, the radiance field was modeled as being independent of \$\phi\$; that is,

$$L(\Theta, \Phi, \theta, t) = \frac{1}{\pi} M(\Theta, \Phi, t) R(\theta)$$
 (3)

where M is the radiant exitance at the top of the atmosphere and  $R(\theta)$  is a directional function assumed to be independent of position and dependent only upon the zenith angle of the exiting ray. The time dependence is removed by considering monthly averages of measurements. Substituting equations (2) and (3) into equation (1) yields

$$\mathbf{m}(\Theta_{\mathbf{S}}, \Phi_{\mathbf{S}}) = \frac{1}{\pi} \int_{\mathbf{FOV}} \mathbf{M}(\Theta, \Phi) \ \mathbf{R}(\theta) \ \cos \alpha \ d\Omega \tag{4}$$

or

$$m(\Theta_S, \Phi_S) = \mathcal{L}[M(\Theta, \Phi)]$$

where  $\mathscr{L}$  denotes the linear integral measurement operator of equation (4). It has been shown by Smith and Green (1976) that the eigenfunctions of this linear operator are spherical harmonics for any  $R(\theta)$ ; that is,

$$\mathcal{L}\left[\mathbf{Y}_{n}^{m}\right]\left(\boldsymbol{\Theta}_{\mathbf{S}},\boldsymbol{\Phi}_{\mathbf{S}}\right) = \lambda_{n} \mathbf{Y}_{n}^{m}\left(\boldsymbol{\Theta}_{\mathbf{S}},\boldsymbol{\Phi}_{\mathbf{S}}\right) \tag{5}$$

where  $Y_n^m(\Theta_s,\Phi_s)$  is a spherical harmonic of order m and degree n evaluated at the subsatellite point. The associated eigenvalue  $\lambda_n$  is given by

$$\lambda_{n} = 2 \int_{\alpha=0}^{\alpha=\alpha_{h}} P_{n}^{0}(\cos \gamma) R(\theta) \cos \alpha \sin \alpha d\alpha$$
 (6)

where  $P_n^0(\cos\gamma)$  denotes the Legendre polynomial of degree n as a function of the Earth central angle  $\gamma$ . (See fig. 5.) The integration is from nadir  $(\alpha=0)$  to  $\alpha_h$ , where  $\alpha_h$  is the cone angle to the horizon. Eigenvalues through degree 12 are listed in table I for both a Lambertian and limb-darkening model (Raschke et al. (1973); Green and Smith (1978)).

Over most of the region subtended by the Nimbus 6 WFOV radiometer,  $g(\alpha)$  has a  $\cos\alpha$  response except for an enhanced response near 60° which is thought to be due to multiple reflections from the ERB instrument. However, since the amount of energy contributed to the measurements from 60° is small, it is felt that modeling the radiometer as a cosine response is adequate.

The effect of choosing one directional function for the entire globe has been investigated. An extreme case was considered where the entire globe was assumed Lambertian, then the entire globe was assumed greatly limb-darkened and the two results were compared. The results reflect the sensitivity of  $R(\theta)$  being independent of position. The two solutions gave identical global means. The area-weighted mean of the absolute zonal difference of the Lambertian solution and the limb-darkened solution was 0.55 W-m<sup>-2</sup>. The area-weighted mean of the absolute  $10^{\circ}$  regional difference of the Lambertian solution and the limb-darkened solution was 0.75 W-m<sup>-2</sup>. The directional function used in the present study (Green and Smith (1978)) is intermediate to the two directional functions discussed above and is considered to yield good results.

### Deconvolution

Because spherical harmonics are eigenfunctions of the measurement operator, it is convenient to represent the radiant exitance at TOA by a truncated series of spherical harmonics; that is,

$$\mathbf{M}(\Theta, \Phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} b_{n}^{m} \mathbf{Y}_{n}^{m}(\Theta, \Phi)$$

Also, let the measurements be represented as

$$m(\Theta, \Phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} a_n^m Y_n^m(\Theta, \Phi)$$

and from equations (4) and (5) it follows that

$$M(\Theta,\Phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} \frac{a_n^m}{\lambda_n} Y_n^m(\Theta,\Phi)$$

where  $a_n^m$  and  $b_n^m$ , in W-m<sup>-2</sup>, are complex coefficients of spherical harmonics.

### Estimation of Coefficients

For the purposes of this analysis, a real formulation of the spherical harmonics was used. Thus, the radiant exitance at the top of the atmosphere is

$$\mathbf{M}(\Theta, \Phi) = \sum_{n=0}^{N} \sum_{m=0}^{n} \lambda_{n}^{-1} \left[ \hat{\mathbf{c}}_{n}^{m} \hat{\mathbf{Y}}_{n}^{m}(\Theta, \Phi) + \hat{\mathbf{s}}_{n}^{m} \hat{\mathbf{Y}}_{n}^{m}(\Theta, \Phi) \right]$$
 (7)

where  $\hat{c}_n^m$  and  $\hat{s}_n^m$  are real coefficients and are estimated from the measurements such that

$$m(\Theta, \Phi) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left[ \hat{C}_{n}^{m} \hat{Y}_{n}^{m}(\Theta, \Phi) + \hat{S}_{n}^{m} \tilde{Y}_{n}^{m}(\Theta, \Phi) \right]$$
(8)

Moreover, the following definitions apply:

$$\hat{\mathbf{Y}}_{\mathbf{n}}^{\mathbf{m}}(\Theta, \Phi) = \mathbf{N}_{\mathbf{n}}^{\mathbf{m}} \cos(\mathbf{m}\Phi) \, \mathbf{P}_{\mathbf{n}}^{\mathbf{m}}(\cos \Theta) \tag{9}$$

$$\widetilde{Y}_{n}^{m}(\Theta,\Phi) = N_{n}^{m} \sin(m\Phi) P_{n}^{m}(\cos\Theta)$$
(10)

$$N_{n}^{m} = \left[ (2n + 1) (n - m)! \frac{\left(2 - \delta_{0}^{m}\right)}{(n + m)!} \right]^{1/2}$$
(11)

$$\int_{\text{Sphere}} \hat{Y}_{n}^{m} \hat{Y}_{h}^{k} d\sigma = \int_{\text{Sphere}} \tilde{Y}_{n}^{m} \tilde{Y}_{h}^{k} d\sigma = 4\pi \delta_{n}^{h} \delta_{m}^{k}$$
(12)

$$\int_{\text{Sphere}} \hat{Y}_{n}^{m} \tilde{Y}_{h}^{k} d\sigma = 0$$
 (13)

Also, the following notation will be convenient:

$$C_{\mathbf{m}}(\Phi_{1},\Phi_{2}) = \int_{\Phi=\Phi_{1}}^{\Phi=\Phi_{2}} \cos(\mathbf{m}\Phi) d\Phi \tag{14}$$

$$S_{\mathbf{m}}(\Phi_{1},\Phi_{2}) = \int_{\Phi=\Phi_{1}}^{\Phi=\Phi_{2}} \sin(\mathbf{m}\Phi) d\Phi$$
 (15)

$$I_{n}^{m}(\Theta_{1},\Theta_{2}) = \int_{\Theta=\Theta_{1}}^{\Theta=\Theta_{2}} P_{n}^{m}(\cos\Theta) \sin\Theta d\Theta$$
 (16)

Multiplying equation (8) by  $\hat{Y}_n^m(\Theta,\Phi)$  and integrating over the sphere, taking into account the orthogonal conditions of equations (12) and (13), gives the coefficients at satellite altitude as a function of the measurements; that is,

$$\hat{C}_{n}^{m} = (4\pi)^{-1} \int_{\text{Sphere}} m(\Theta, \Phi) \hat{Y}_{n}^{m}(\Theta, \Phi) d\sigma$$
 (17)

However, the measurements  $m(\Theta,\Phi)$  are not known everywhere but only at discrete points, so that equation (17) cannot be applied directly. This problem was overcome by dividing the sphere into grid regions and assuming  $m(\Theta,\Phi)$  was constant over each region. Thus,  $m_k$  denotes the average of all measurements that are in the kth region. The coefficients in equation (17) are given by

$$\hat{C}_n^m = (4\pi)^{-1} \sum_{k=1}^K m_k \int_{\text{Region } k} \hat{Y}_n^m(\Theta, \Phi) d\sigma$$

and from equation (9)

$$\hat{C}_{n}^{m} = N_{n}^{m}(4\pi)^{-1} \sum_{k=1}^{K} m_{k} \int_{\Phi=\Phi_{1}(k)}^{\Phi=\Phi_{2}(k)} \cos\left(m\Phi\right) d\Phi \int_{\Theta=\Theta_{1}(k)}^{\Theta=\Theta_{2}(k)} P_{n}^{m}(\cos\Theta) \sin\Theta d\Theta$$

and from equations (14) and (16)

$$\hat{C}_{n}^{m} = N_{n}^{m} (4\pi)^{-1} \sum_{k=1}^{K} m_{k} C_{m}(k) I_{n}^{m}(k)$$
(18)

Similarly,

$$\hat{S}_{n}^{m} = N_{n}^{m} (4\pi)^{-1} \sum_{k=1}^{K} m_{k} S_{m}(k) I_{n}^{m}(k)$$
(19)

The regions as defined in this analysis are approximately equal area regions, the sides of which are latitude and meridian lines. At the equator the regions are square. For  $5^{\circ}$  by  $5^{\circ}$  regions, the sphere is divided into  $5^{\circ}$  latitudinal bands and each band is divided into an integral number of areas such that each area has approximately the same area. The band from  $0^{\circ}$  to  $5^{\circ}$  colatitude contains three regions. The band from  $5^{\circ}$  to  $10^{\circ}$  colatitude contains nine regions, etc., down to the band from  $85^{\circ}$  to  $90^{\circ}$  colatitude, which contains 72 square regions,  $5^{\circ}$  by  $5^{\circ}$ . The total number of regions for a  $5^{\circ}$  by  $5^{\circ}$  grid system is K = 1654 for the entire globe.

Equations (18) and (19) provide a very efficient way to determine the spherical harmonic coefficients of the measurement representation. First, the total number of measurements is greatly reduced by averaging all measurements in the same region. The integrals  $C_{\rm m}$  and  $S_{\rm m}$  are rapidly computed recur-

sively with the trigonometric addition formulas. Recursive formulas for  $I_n^m$  are given in the appendix. The computational effort is halved by taking advantage of the symmetry of the grid system about the equator. If k' corresponds to the region that is the reflection about the equator of the kth region,

then it can be shown that  $I_n^m(k') = (-1)^{n+m} I_n^m(k)$ . Thus, k in equations (18) and (19) need only range over the regions in the Northern Hemisphere, and

$$\hat{C}_{n}^{m} = N_{n}^{m} (4\pi)^{-1} \sum_{k=1}^{K/2} \left[ m_{k} C_{m}(k) I_{n}^{m}(k) + m_{k} C_{m}(k') I_{n}^{m}(k') \right]$$
(20)

or

$$\hat{C}_{n}^{m} = N_{n}^{m} (4\pi)^{-1} \sum_{k=1}^{K/2} \left[ m_{k} + (-1)^{n+m} m_{k} \right] C_{m}(k) I_{n}^{m}(k)$$

Similarly,

$$\hat{S}_{n}^{m} = N_{n}^{m} (4\pi)^{-1} \sum_{k=1}^{K/2} \left[ m_{k} + (-1)^{n+m} m_{k} \right] S_{m}(k) I_{n}^{m}(k)$$
(21)

The spherical harmonic coefficients of the measurement representation at satellite altitude are computed using equations (20) and (21). The coefficients for the top of the atmosphere are given by

$$c_n^m = \frac{\hat{c}_n^m}{\lambda_n} \tag{22}$$

and

$$s_n^m = \frac{\hat{s}_n^m}{\lambda_n} \tag{23}$$

where  $\lambda_n$  is the nth eigenvalue from equation (6) based on the limb-darkening function  $R(\theta)$ . When referring to the time-dependent nature of these coefficients,  $C_n^m(t)$  and  $S_n^m(t)$  are used.

### RESULTS AND DISCUSSION

One year of longwave radiation data from the ERB instrument aboard Nimbus 6 and taken during the period July 1975 through June 1976 has been analyzed to estimate the longwave radiant exitance at the top of the atmosphere (TOA). The radiative field was represented by a spherical harmonic expansion, and the coefficients were estimated with measurement data.

### Degree Variance

The spherical harmonic coefficients competed using equations (22) and (23) can be used to define the spatial spectrum of the radiation field in terms of degree variance. From Green and Smith (1978), degree variance for a given degree n at satellite altitude is defined to be

$$\sigma_n^2 = \sum_{m=0}^n \left[ \left( \hat{\mathbf{c}}_n^m \right)^2 + \left( \hat{\mathbf{s}}_n^m \right)^2 \right]$$

At the top of the atmosphere (TOA), the degree variance for a given degree  $\,$  n is

$$\sigma_n^2 \Big|_{TOA} = \left(\frac{\sigma_n}{\lambda_n}\right)^2$$

The degree variance for all n is analogous to the amplitude spectrum in ordinary harmonic analysis and gives an indication of the power in each spatial frequency. Degree variance plots for each month as a function of degree n at satellite altitude and at TOA are used to establish the limit in degree to which a spherical harmonic representation of the data can be realized. Figure 6 shows the degree variance plot for the average of the 12-month data set for each degree 1 through 30. The lower curve is the degree variance at satellite altitude, and the upper curve represents the degree variance at TOA. Divergence of the two curves begins at about degree 12.

Figure 7, similar to figure 6, shows the standard deviation of average degree variance over 12 months at the top of the atmosphere for degree 1 to

degree 30. During the year there are 12 values of  $\sigma_n^2$  for each degree n. Standard deviation of average degree variance at satellite altitude is similar to the variation at the top of the atmosphere.

The degree at which the spherical harmonic expansion should be truncated impacts the uncertainty of the radiant exitance contour map solutions. Based on a 12-month data set, figure 6 shows the resolution limit of the spherical harmonic representation to be about 12th degree, with the spectral region of higher frequency components beyond 12th degree produced mainly by noise comprised of variations in the radiation field during the month and measurement errors. Monthly sampling does not appear to affect the 12th degree limit on truncation. This can be seen by considering the LW radiation data for March 1976, for which the time average of radiant exitance covered only the second half of the month because of some operational problems. However, the resolution limit was still 12th degree based on the degree variance plot for March.

Spatial sampling of the data for August 1975 averaged over the month for  $5^{\circ}$  by  $5^{\circ}$  grid regions and then for  $2\frac{10}{2}$  by  $2\frac{1}{2}$  grid regions showed that the resolution limit did not change when grid size was changed.

Attempting to represent the radiation field beyond the resolution limit of the sensor leads to incorrect values of radiant exitance at TOA because the noise is being amplified by the eigenvalues  $\lambda_n$ .

To give some idea of how sensitive the solution is to the degree of truncation, the measurements for the month of August 1975 were carried out to a 15th degree expansion and then compared with a 12th degree expansion for August, both for a Lambertian model. Results show no difference in global mean, pole-to-pole gradient, and equator-to-pole gradient. The area-weighted mean of absolute zonal differences of 15th degree minus 12th degree was 0.85 W-m<sup>-2</sup>. The area-weighted mean of absolute 10° grid regional differences was 3.94 W-m<sup>-2</sup>.

### Spherical Harmonic Coefficients

Spherical harmonic coefficients for 12 months of longwave radiation data were computed using equations (22) and (23) which are based on the monthly mean value of the measurements in each of the 1654 grid regions. Results are for a degree 12 spherical harmonic expansion. For a 12th degree expansion, 169 coefficients are required to specify the radiation field.

For a 12th degree spherical harmonic expansion approximately 80 percent of total degree variance is accounted for by the 12 axisymmetric coefficients  $c_1^0$  to  $c_{12}^0$ . The first five axisymmetric coefficients represent about 70 percent of the summed degree variance. The axisymmetric terms for the 12 months of Nimbus 6 longwave radiation data are listed in table II. Nonaxisymmetric terms through degree 12 and through order 12 for the 12 months are shown in table III.

Terms below the indicated stairstep are cosine terms  $C_n^m$ ; terms above the stairstep are sine terms  $s_n^m$ . Units of all terms are in W-m<sup>-2</sup>.

### Geographical Distribution of Longwave Radiation

Radiation contour maps for each month are shown in figures 8(a) to 8(1). Figure 9 shows monthly averages of zonal emitted radiant exitance for each of the 12 months. In essence, figure 9 shows a profile of each contour map where radiant exitance has been averaged over longitude at each 50 latitude. The patterns in the maps (figs. 8(a) to 8(l)) are mainly influenced by surface temperature, by the mean cloud cover and cloud height, and by the temperature lapse rate and moisture content of the atmosphere. These maps show the geographical distribution of emitted longwave radiant exitance for the 12-month period and are based on a degree 12 spherical harmonic representation of the

radiation field. It can be seen from the contour maps, from figure 9, and from the table of spherical harmonic coefficients (table II) that the radiation field for each month is zonal in character and especially well marked in the Southern Hemisphere.

The contour maps also define the variation of major features for each Centers of high and low radiant exitance show some shifting in position throughout the year. Highs are generally located in the tropical and subtropical regions between -350 latitude and +350 latitude. A narrow band of lows in temperate and low latitudes between -100 and +200 appears throughout the year. These lows appear over areas with high cyclonic activity. Lows extend to high latitudes in both hemispheres, with the extension being a dominant feature in the Northern Hemisphere. Noteworthy are the lows over Greenland, Siberia, Antarctica, Central Africa, and Southeast Asia, the highs over Northern and Southern Africa, and the high over Australia. The lows over Antarctica show up during each month and are presumably picking up the Antarctica ice cap. emission throughout the year occurs over Antarctica. This low shows some shifting and variability. Emission over Antarctica is in step with the seasons, increasing from September through March and decreasing during the next 6 months. Greenland, the Beaufort Sea off the coast of Alaska, Siberia, and the Siberian Sea with its thick ice shield show the lowest emission over the Arctic, but not as low as the emission over Antarctica. In terms of temperature, the Arctic is much warmer on the average throughout the year than Antarctica.

Another area of low emission is Southeast Asia during the monsoon summer, when the amount of longwave radiation emitted over Southeast Asia is less than the emitted radiation over Greenland during the same time period. This is due to monsoon clouds being relatively opaque with surfaces that are usually high and cold. During the winter months the emitted radiation over Southeast Asia increases because the winter monsoons bring dry, clear weather for several months.

The low over Central Africa appears during each month and shows very little movement. This low is located in the vicinity of the Congo Basin and the Equatorial rain forests, which are characterized by continuously warm weather and abundant rainfall. During the summer season, highs exist over Northern and Southern Africa in the vicinity of the Sahara and Kalahari Deserts. These highs show some movement with the seasons. There is also a high that appears consistently near the western coast of the Australian desert region. This high shows some movement with the seasons.

The monthly contour maps were shown to have a 12th order spherical harmonic expansion. The sensitivity caused by changing the order of expansion shows up clearly in the contour maps. Green and Smith (1978) analyzed the Nimbus 6 long-wave data for August 1975 but only produced a 5th order spherical harmonic expansion. Their contour map for August is qualitatively similar to the August contour map from the present analysis. However, the two maps differ in both location and spatial extent of high and low areas of radiation. In effect, the 5th order expansion for the month of August does not contain as much high frequency information of the radiation field as does the 12th order expansion.

### Time Histories of Zonal Coefficients

Examination of the  $C_n^m(t)$  and  $S_n^m(t)$  presented in table II and table III shows that the axisymmetric terms (m = 0) dominate up to n = 5 with about 70 percent of the degree variance in the first five zonal coefficients. Some of these coefficients have obvious physical meanings.

Figure 10 shows the yearly cycle of the global average  $C_0^0(t)$  of emitted

longwave radiation. The plot of  $C_0^0(t)$  represented by the square symbols in figure 10 is the only term from the present analysis that can be readily compared with global average values of emitted radiation from Nimbus 6 published by other sources, such as Ellis et al. (1978) and Jacobowitz et al. (1979). In making the comparisons, TOA is at 30 km above the Earth's surface in all cases, and the radiant exitant values are averages of daytime and nightime measurements. All data sets incorporated a calibration correction referred to by Smith et al. (1977). Both Ellis and Jacobowitz used the inverse square approximation to determine radiant exitance at TOA. All three data sets show the same trend in global average for the year. However, the global average of radiant exitance based on the work by Ellis et al. (1978) is about  $4 \text{ W-m}^{-2}$  higher (on the average) than the global average of radiant exitance from the present work. From January through June the global average from Jacobowitz et al. (1979) compares

very well with the global averages  $C_0^0(t)$ . Also included in figure 10 is a 29-month data set by Ellis et al. (1978) of global average. This data set is comprised of a number of sources. The trend is very irregular, with unusually high values of gobal radiant exitance in April and May. Some differences in global averages as reported by the different investigators (fig. 10) may be due to the way data were edited before being analyzed. Calibration corrections may have also been different.

The  $C_1^0(t)$  term in figure 11 is a measure of hemispherical or pole-to-pole differences. Its annual cycle is seen to have a nearly perfect sine shape, with a total range between its minimum and maximum values of about 20 W-m<sup>-2</sup>. It does not oscillate about zero but has a bias of 2.5 W-m<sup>-2</sup>. If the Earth is symmetric

about the equator, one would expect the  $\, c_n^0 \,$  terms for odd  $\, n \,$  values to have time histories which are symmetric about the time axis. Thus, the 2.5 W-m<sup>-2</sup>

bias in the annual cycle of  $C_1^0(t)$  is presumed to be due to land and ocean distributional differences between the Northern and Southern Hemispheres. The

 $C_2^0(t)$  term in figure 11 may be considered to be a measure of equator-to-pole gradient. It is seen to have an average value of -25.6 W-m<sup>-2</sup>, with a small variation.

The  $C_n^0(t)$  terms for n=3 to n=12 are shown in figure 12. The  $C_3^0$  term is nearly sinusoidal, with a total variation of 15 W-m<sup>-2</sup> and a mean of 4 W-m<sup>-2</sup>. As with  $C_1^0$ , this bias is presumed to be the result of hemispheric

differences of land and ocean distribution. The  $C_4^0$  term has a mean of approximately -7 W-m<sup>-2</sup> with variation of 5 W-m<sup>-2</sup>. Its shape is not so sinusoidal as the  $C_1^0$  or  $C_3^0$  histories. The  $C_5^0$  to  $C_{10}^0$  terms each have a significant annual sine component, and the  $C_6^0$  and  $C_8^0$  terms have a bias such that they do not change signs. The maximum absolute value decreases with increasing n until  $C_{11}^0$  and  $C_{12}^0$ , which are small and show little discernible pattern.

Whether the lack of pattern in the computed values of  $C_{11}^0$  and  $C_{12}^0$  is due to the nature of the atmosphere or due to the limitations of sampling and analysis is unclear at present.

### Effect of Grid System

The reason for using the grid-system averaging technique is to reduce the computer computational burden. This is done at the expense of smoothing the data. One way to assess the effect of grid systems on the present analysis, which is based on a 5° by 5° grid system, is to compare the degree variance plots based on a different size grid system. This comparison was done for the

month of August 1975 for a  $5^{\circ}$  and a  $2^{-}$  grid system. The computational effort is 4 times greater for the  $2^{-}$  grid system. The results are shown in figure 13.

Out to degree 7, the degree variance plots for the two grid systems are the same. From degree 7 to degree 12, where their contribution is small, the degree variances differ only by a small amount. Thus, based on comparing degree variances, no significant improvement is gained in going from a 5° by 5° grid system

to a  $2\frac{10}{2}$  by  $2\frac{10}{2}$  grid system.

### CONCLUSIONS AND RECOMMENDATIONS

Based upon the results of the analysis of the first year of data from the Nimbus 6 WFOV radiometer, the following conclusions regarding Earth emitted radiation are drawn:

1. Degree variance plots for 12 months of LW radiation data show that the limit for a spherical harmonic representation of the Nimbus 6 WFOV LW data is 12th degree. The degree variance plots also reveal that most of the power is in the lower degree terms. The axisymmetric (zonal) terms dominate, with their coefficients representing approximately 80 percent of the degree variance for a 12-month period.

- (2) Contour maps of the radiation field are computed. These maps show the geographical distribution of Earth emitted radiant exitance for 12 months. The maps reveal distinct highs and lows and their shift with the seasons. Highs are generally located between  $-35^{\circ}$  latitude and  $+35^{\circ}$  latitude. A band of lows is located between  $-10^{\circ}$  latitude and  $+20^{\circ}$  latitude. Lows extend to high latitudes in both hemispheres, with the extension being a dominant feature in the Northern Hemisphere.
- (3) Some of the spherical harmonic coefficients have physical meanings; for example,  $C_0^0(t)$  is the global average emitted radiation,  $C_1^0(t)$  represents the pole-to-pole gradient, and  $C_2^0(t)$  is a measure of the equator-to-pole gradient. Comparison of  $C_0^0(t)$  from the present study with the global average of emitted radiation computed by Ellis et al. (J. Geophys. Res., vol. 83, no. C4, Apr. 20, 1978) shows that  $C_0^0(t)$  is about 4 W-m<sup>-2</sup> lower than their global average, although the annual trend is the same. The global average from Jacobowitz et al. (J. Atmos. Sci., vol. 36, no. 3, Mar. 1979) shows better agreement with  $C_0^0(t)$ . The  $C_1^0(t)$  term has a bias of about 2.5 W-m<sup>-2</sup>, presumably due to distributional differences between land and ocean in the Northern and Southern Hemispheres.
- 4. Based on comparing degree variances, no significant improvement is gained in going from a  $5^{\circ}$  by  $5^{\circ}$  grid system to a  $2^{-}$  by  $2^{-}$  grid system. Furthermore, the averaging of data over  $5^{\circ}$  by  $5^{\circ}$  regions is a reasonable approach.

Subsequent years of data from the Nimbus 6 ERB are needed in order to establish a longer term average of the annual cycle and to provide a basis for studies of interannual variability. Nonaxisymmetric terms need to be analyzed for annual variation. Longitudinal spectra of emitted radiation as a function of latitude need to be computed in order to gain insight into processes causing a nonaxisymmetric distribution.

Langley Research Center National Aeronautics and Space Administration Hampton, VA 23665 October 6, 1980

#### · APPENDIX

### INTEGRATION OF ASSOCIATED LEGENDRE POLYNOMIALS

The problem is to determine recursive formulas for  $I_{n}^{m}$ , where

$$I_{n}^{m}(\Theta_{1},\Theta_{2}) \equiv \int_{\Theta=\Theta_{1}}^{\Theta=\Theta_{2}} P_{n}^{m}(\cos\Theta) \sin\Theta d\Theta$$
(A1)

where  $P_n^{m}(\cos\Theta)$  is an associated Legendre polynomial of degree n and

order m and  $\Theta$  is colatitude. Recursive formulas for  $I_n^m$  are given by Fricke (1978). The purpose of this appendix is to expand the derivation and to express the results in a form compatible with this paper.

Equations (A2) and (A3) are obtained from Korn and Korn (1961) as follows (eq. (A2) is equivalent to eq. (21.8-55) in the reference):

$$(n-m+1) P_{n+1}^{m}(\cos\Theta) = (2n+1) \cos\Theta P_{n}^{m}(\cos\Theta) - (n+m) P_{n-1}^{m}(\cos\Theta)$$

$$(0 \le m \le n-1) \quad (A2)$$

$$\sin^2\Theta\,\frac{d}{d(\cos\Theta)}\left[P_n^m(\cos\Theta)\right] \,=\, (n+1)\,\cos\Theta\,\,P_n^m(\cos\Theta)\,\,-\,\,(n-m+1)\,\,P_{n+1}^m(\cos\Theta)$$
 
$$(0\,\leq\,m\,\leq\,n) \qquad (A3)$$

Integration of equation (A2) using equation (A1) gives

$$I_{n+1}^{m}(\Theta_{1},\Theta_{2}) = (2n+1)(n-m+1)^{-1} \int_{\Theta_{1}}^{\Theta_{2}} P_{n}^{m}(\cos\Theta) \cos\Theta \sin\Theta d\Theta$$

$$- (n+m)(n-m+1)^{-1} I_{n-1}^{m}(\Theta_{1},\Theta_{2})$$
(A4)

Integration of equation (A3) gives

$$\int_{\Theta_1}^{\Theta_2} \sin^2\Theta \; \frac{\mathrm{d}}{\mathrm{d}(\cos\Theta)} \Big[ \mathtt{P}_n^{m}(\cos\Theta) \Big] \; \sin\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \sin\Theta \; \cos\Theta \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathtt{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \int_{\Theta_1}^{\Theta_2} \; \mathrm{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \mathrm{P}_n^{m}(\cos\Theta) \; \mathrm{d}\Theta \; = \; (n+1) \; \mathrm{P}_n^{m}(\Theta) \; = \; (n+1) \; \mathrm{$$

- 
$$(n - m + 1)$$
  $\int_{\Theta_1}^{\Theta_2} P_{n+1}^m(\cos\Theta) \sin\Theta d\Theta$  (A5)

Integration of the left-hand side of equation (A5) by parts gives

$$(n-1) \int_{\Theta_1}^{\Theta_2} P_n^{\mathsf{m}}(\cos\Theta) \cos\Theta \sin\Theta d\Theta = -\sin^2\Theta P_n^{\mathsf{m}}(\cos\Theta) \Big|_{\Theta_1}^{\Theta_2} + (n-m+1) I_{n+1}^{\mathsf{m}}(\Theta_1,\Theta_2)$$
 (A6)

Elimination of the integral between equations (A4) and (A6) and collecting terms gives the desired recursion equation

$$\mathbf{I}_{n+1}^{m}(\Theta_{1},\Theta_{2}) = (n+2)^{-1}(n-m+1)^{-1} \left[ (2n+1) \sin^{2}\Theta P_{n}^{m}(\cos\Theta) \middle|_{\Theta_{1}}^{\Theta_{2}} + (n-1)(n+m) \mathbf{I}_{n-1}^{m}(\Theta_{1},\Theta_{2}) \right] \\
\left( m = 0, 1, 2, \ldots \\
n = m+1, m+2, \ldots \right) \tag{A7}$$

The recursion equation (A7) for  $I_{n+1}^m$  requires two initial values, namely,  $I_m^m$  and  $I_{m+1}^m$ . Equation (A8) is obtained from Korn and Korn (1961) as follows (eq. (A8) is equivalent to eq. (21.8-54) in the reference):

$$P_m^{\rm m}(\cos\Theta) = (1)(3)(5) \dots (2m-1) \sin^{\rm m}\Theta \qquad (m=1, 2, \dots)$$
 (A8)

and from the definition of  $I_m^m$  (eq. (A1)),

$$I_{m}^{m}(\Theta_{1},\Theta_{2}) = \int_{\Theta_{1}}^{\Theta_{2}} (1)(3)(5) \dots (2m-1) \sin^{m+1} \Theta d\Theta$$
 (A9)

By integration,

Substituting from equations (A8) and (A9) gives

$$I_{m}^{m}(\Theta_{1},\Theta_{2}) = -(m+1)^{-1} \cos \Theta P_{m}^{m}(\cos \Theta) \Big|_{\Theta_{1}}^{\Theta_{2}} + m(2m-1)(2m-3)(m+1)^{-1} I_{m-2}^{m-2}(\Theta_{1},\Theta_{2}) \Big|_{\Theta_{1}}^{m-2} (m=2, 3, ...) \quad (A10)$$

Evaluating equation (A3) at n = m gives

$$\sin^2\Theta \frac{d}{d(\cos\Theta)} \left[ P_m^m(\cos\Theta) \right] = (m + 1) \cos\Theta P_m^m(\cos\Theta) - P_{m+1}^m(\cos\Theta)$$
 (All)

Differentiating  $P_{m}^{m}$  of equation (A8) and simplifying yields the result

$$P_{m+1}^{m}(\cos\Theta) = (2m+1)\cos\Theta P_{m}^{m}(\cos\Theta)$$
 (m = 0, 1, . . .) (A12)

Thus, equation (AlO) can be simplified as

$$I_{m}^{m}(\Theta_{1},\Theta_{2}) = -(2m+1)^{-1}(m+1)^{-1}P_{m+1}^{m}(\cos\Theta) \Big|_{\Theta_{1}}^{\Theta_{2}} + m(2m-1)(2m-3)(m+1)^{-1}I_{m-2}^{m-2}(\Theta_{1},\Theta_{2}) \Big|_{\Theta_{1}}^{m-2}$$

$$(m=2,3,\ldots) \quad (A13)$$

To start this recursion, the values  $I_0^0$  and  $I_1^1$  are needed, or

$$I_0^0(\Theta_1,\Theta_2) = \int_{\Theta_1}^{\Theta_2} (1) \sin \Theta d\Theta = -\cos \Theta \Big|_{\Theta_1}^{\Theta_2}$$

$$(A14)$$

$$\mathbf{I}_{1}^{1}(\Theta_{1},\Theta_{2}) = \int_{\Theta_{1}}^{\Theta_{2}} (\sin\Theta) \sin\Theta d\Theta = \frac{1}{2}(\Theta - \sin\Theta \cos\Theta) \Big|_{\Theta_{1}}^{\Theta_{2}}$$
(A15)

Next, an expression for  $I_{m+1}^m$  is derived. From equations (A12) and (A8),

$$\int_{\Theta_{1}}^{\Theta_{2}} P_{m+1}^{m}(\cos\Theta) \sin\Theta \ d\Theta = (1)(3)(5) \dots (2m-1)(2m+1) \int_{\Theta_{1}}^{\Theta_{2}} \sin^{m+1}\Theta \cos\Theta \ d\Theta$$

or

$$I_{m+1}^{m}(\Theta_{1},\Theta_{2}) = (1)(3)(5) \dots (2m-1)(2m+1)(m+2)^{-1} \sin^{m+2}\Theta \Big|_{\Theta_{1}}^{\Theta_{2}}$$

Simplifying with equation (A8) gives

$$I_{m+1}^{m}(\Theta_{1},\Theta_{2}) = (m+2)^{-1}(2m+3)^{-1}P_{m+2}^{m+2}(\cos\Theta)\Big|_{\Theta_{1}}^{\Theta_{2}} \qquad (m=0,1,2,\ldots)$$
 (A16)

Thus, in summary, the five recursive formulas for  $\ensuremath{\text{I}}_n^m$  are

$$I_0^0(\Theta_7,\Theta_2) = -\cos\Theta \begin{vmatrix} \Theta_2 \\ \Theta_1 \end{vmatrix}$$
(A14)

$$I_{1}^{1}(\Theta_{1},\Theta_{2}) = \frac{1}{2}(\Theta - \sin\Theta \cos\Theta) \Big|_{\Theta_{1}}^{\Theta_{2}}$$
(A15)

$$I_{m}^{m}(\Theta_{1},\Theta_{2}) = -(2m+1)^{-1} (m+1)^{-1} P_{m+1}^{m}(\cos\Theta) \begin{vmatrix} \Theta_{2} \\ \Theta_{1} \end{vmatrix} + m(2m-1)(2m-3)(m+1)^{-1} I_{m-2}^{m-2}(\Theta_{1},\Theta_{2})$$

$$(m=2, 3, \ldots) \qquad (A13)$$

### APPENDIX

$$I_{m+1}^{m}(\Theta_{1},\Theta_{2}) = (m+2)^{-1}(2m+3)^{-1}P_{m+2}^{m+2}(\cos\Theta)\Big|_{\Theta_{1}}^{\Theta_{2}} \qquad (m=0,1,2,\ldots)$$
 (A16)

$$I_{n+1}^{m}(\Theta_{1},\Theta_{2}) = (n+2)^{-1}(n-m+1)^{-1} \left[ (2n+1) \sin^{2}\Theta P_{n}^{m}(\cos\Theta) \middle|_{\Theta_{1}}^{\Theta_{2}} + (n-1)(n+m) I_{n-1}^{m}(\Theta_{1},\Theta_{2}) \right]$$

$$\begin{pmatrix} m=0,1,2,\ldots\\ n=m+1,m+2,\ldots \end{pmatrix} \tag{A7}$$

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TABLE I.- EIGENVALUES OF MEASUREMENT OPERATOR

 $[r_e = 6408.165 \text{ km}; h = 1070 \text{ km}]$ 

	$\lambda_{n}$	for -
n	Lambertian model	Limb-darkening model
0	0.7343	0.7343
1	.7217	.7232
2	.6975	.7014
3	.6632	.6704
14	.6208	.6317
5	.5726	.5873
6	.5214	.5393
7	.4693	.4899
8	.4185	.4408
9	. 3707	. 39 36
10	. 3267	. 3494
11	.2874	.3091
12	.2526	.2728

TABLE II.- SPHERICAL HARMONIC (ZONAL) COEFFICIENTS FOR 12 MONTHS OF NIMBUS 6 LONGWAVE RADIANT EXITANCE (W-m<sup>-2</sup>) THROUGH 12TH DEGREE AT TOP OF ATMOSPHERE<sup>a</sup>

	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
c <sub>0</sub>	235.042	235.663	232.706	230.708	228.412	228.963	229.931	229.285	231.055	230.793	232.395	238.222
c <sub>1</sub> 0	12.501	11.287	8.171	4.548	-1.003	-5.662	-7.707	-7.817	-4.067	151	5.639	11.439
c <sub>2</sub> <sup>0</sup>	-21.222	-23.354	-25.455	-27.252	-27.313	-26.577	-26.026	-27.664	-27.348	-26.238	-26.131	-25.549
c <sub>3</sub> 0	9.966	12.091	7.212	3.856	-1.040	-3.190	-3.204	-2.172	1.907	5.853	8.656	9.044
C <sub>4</sub> O	-8.960	-9.304	-8.464	-7.813	-4.577	-3.792	-4.475	-5.226	-5.678	-5.637	-5.753	<b>-7.</b> 267
c <sub>5</sub> 0	-4.200	-5.206	-3.027	370	• 4 2 3	2.394	3.798	6.361	6.855	3.059	658	-2.626
c <sub>6</sub>	2.719	2.933	4.697	5.245	6.942	7.414	7.550	7.841	5.643	3.167	1.321	4.151
c <sub>7</sub> 0	7.528	9.353	6.928	5.132	•506	-3.346	-4.715	-4.899	-1.912	•003	5.156	6.946
c <sub>8</sub> 0	-5.707	-4.183	-3.671	-1.914	-4.612	-4.349	-3.440	-3.244	-7.136	-7.810	-6.087	-5.554
c <sub>9</sub> 0	-2.440	-2.964	-3.988	-4.177	•228	3.424	3.600	3.751	1.645	.339	-1,468	-3.243
C <sub>10</sub>	.101	-1.419	1.218	.908	•464	.055	-1.504	1.926	4.108	3.969	1.781	1.566
c <sub>11</sub>	1.448	•442	•618	1.283	1.975	1.039	• 3 98	1.338	930	.857	2.639	1.254
C <sub>12</sub>	•825	000	1.436	1.171	<b></b> 085	1.223	•314	-1.545	462	466	-2.022	.489
				L	L	L	اـــــــا		l	l	L	<b>.</b>

aFor limb-darkening model.

TABLE III.- SPHERICAL HARMONIC (NONAXISYMMETRIC) COEFFICIENTS OF LONGWAVE RADIANT EXITANCE (W-m-2)

Cosine terms are below stairstep and sine terms are above; coefficients are for 12th degree expansion at top of atmosphere; results are for limb-darkening model

(a) July 1975

	12	11	10	9	8	7	6	5	4	3	2	1	m n
	.561	•013	.112	842	305	130	.329	697	-1.243	205	•166	660	12
1	2.996	078	-1.129	.776	-1.047	630	.021	.263	1.149	103	•568	•356	11
2	4.639	4.104	-1.109	.923	045	951	.518	006	1.620	1.066	1.576	1.215	10
3	1.706	2.954	971	031	222	.651	1.267	398	146	1.114	.342	538	9
4	554	-1.382	-2.284	-2.923	-1.386	1.498	1.105	.012	-1.769	931	-1.436	•224	8
5	-5.142	-3.437	.325	-2.242	-1.140	210	420	215	-1.372	-1.946	-1.981	.101	7
6	-2.326	-1.809	2.236	193	284	1.871	817	.121	-1.136	.756	.745	.895	6
7	1.706	• 367	.403	.530	•590	2.045	.746	-1.062	1.058	2.731	• 5 46	-3.562	5
8	1.199	1.988	-1.799	1.527	.527	•727	1.018	2.431	•472	<b></b> 588	•598	3.352	4
9	•451	2.770	.034	.212	359	• 250	061	045	422	-1.552	3.266	3.469	3
10	.318	1.067	1.373	335	309	263	.905	-1.499	•411	923	5.222	<b></b> 63£	2
11	277	-2.851	.091	126	362	.274	.508	-1.480	268	-,472	• 409	-2.953	1
12	.194	-1.889	-1.180	.849	•177	•262	• 29 <u>8</u>	324	.204	525	. 299	• 596	
n	1	2	3	4	5	6	7	8	9	10	11	12	

## TABLE III.- Continued

(b) August 1975

	12	11	10	9	8	7	6	5	4	3	2	1	m/n
	103	158	202	.327	233	•375	.044	• 150	596	1 54	•493	.003	12
1	3.173	.837	351	•329	072	527	•349	. 762	1.166	•085	466	•227	11
2	3.586	4.425	.043	062	.673	187	•493	434	.965	.463	. 893	2.476	10
3	1.080	2.713	1.124	524	.835	.487	•90 <del>6</del>	859	042	.421	• 222	352	9
4	-1.134	-2.459	-1.078	-2.795	.156	.151	204	804	-1.090	186	115	.319	8
5	-3 • 40 4	-2.701	•230	-1.218	-1.664	• 292	-1 •0 90	.023	181	-1.678	067	664	7
. 6	144	075	2.057	• 529	•960	1.050	-1.414	.513	.510	•585	576	-1.014	6
7	1.982	• 590	.853	.884	.749	.321	1.503	-1.217	204	2.848	848	-3.639	5
8	345	1.713	-1.306	.618	847	.787	1.701	•689	133	303	•550	3.993	4
9	855	1.099	-1.443	245	030	.100	•194	835	1.051	-1.437	2.803	4.431	3
10	•116	.637	.841	255	.655	337	357	-1.099	1.179	•095	3.077	-1.380	2
11	. 494	-1.913	1.054	.323	289	013	036	987	260	369	.112	-3.134	1
12	1.564	-1.891	328	.519	091	•433	136	.182	.401	.879	• 539	•729	
n	1	2	3	4	5	6	7	8	9	10	11	12	

TABLE III.- Continued

## (c) September 1975

	12	11	10	9	8	7	6	5	4	3	2	1	m
	021	323	.126	181	260	•012	.682	878	718	.739	104	.104	12
1	1.414	010	277	•009	556	.183	401	096	.221	.155	586	•246	11
2	2.903	4.495	.672	054	593	084	628	.635	.681	076	•029	1.696	10
3	.744	2.165	.783	• 300	.043	.783	.900	.152	.214	.781	.041	-1.894	9
4	439	-2.707	475	-4.352	357	134	• 79 5	315	784	.114	.278	•430	8
5	-2.953	-1.898	•496	-1.527	-2.711	•121	.322	• 798	507	-1.372	144	1.631	7
6	.044	.882	1.931	•493	.179	1.648	352	.601	.380	.775	042	1.229	6
7	1.571	440	283	360	1.091	1.629	1.680	-1.394	905	2.233	159	-3.473	5
8	654	369	-1.030	435	858	• 598	•511	1.301	-1.312	236	109	2.545	. 4
9	521	1.373	•342	• 0 72	898	307	-1.212	041	•300	369	3.223	2.695	3
10	.402	.821	. 68 2	298	017	338	550	-1.693	648	905	4.746	526	2
11	243	-1.335	613	•302	•382	.241	.013	176	922	763	.062	-2.551	1
12	. 5 28	559	482	•902	003	•169	.101	.013	293	554	•219	•525	
n	, 1	2	3	4	5	6	7	8	9	10	11	12	

TABLE III.- Continued

## (d) October 1975

	12	11	10	9	8	7	6	5	4	3	2	1	m/n	.1
	.087	151	• 553	.317	233	•152	300	• 049	•059	•403	734	190	12	
1	3.357	•453	.693	<b></b> 766	713	•414	-1.005	• 363	533	419	407	•476	11	
2	2.737	3.019	.858	289	401	000	547	1.141	•438	237	.943	1.402	10	
3	026	2.617	214	487	468	583	.381	-1.540	•588	1.385	1.209	-1.922	9	
4	-1.145	-2.922	307	-1.557	•107	.019	•599	-1.062	774	1.238	•515	528	8	
5	-2.286	-1.679	352	•312	830	1.229	•129	1.382	•113	-2.218	911	•922	7	,
6	.832	2.848	1.135	1.217	.286	1.499	-1.080	593	2.246	.101	587	1.801	6	S
7	1.905	•525	•202	1.017	1.339	•601	1.636	-1.924	-1.229	1.394	582	-2.329	5	
8	632	•194	290	•034	310	149	437	•319	-3.324	115	-2.134	1.229	4	
9	925	.819	•513	755	943	444	•058	•507	•329	.874	2.509	•543	3	
10	.652	762	• 093	441	•196	683	•066	-1.998	867	215	6.525	•150	2	
11	364	-1.739	648	•213	•729	101	•981	040	• 456	136	087	-1.091	1	
12	.831	•391	•191	• 959	682	•002	•556	• 395	•426	1.509	•299	• 776		
n m	1	2	3	4	5	6	7	8	9	10	11	12		

TABLE III .- Continued

## (e) November 1975

	12	11	10	9	8	7	6	5	4	3	2	1	m
	•975	104	•134	•739	194	•424	•311	895	•019	1.052	-1.096	.075	12
1	1.504	.826	241	030	•014	.151	117	269	840	622	768	1.657	11
2	1.887	•772	•508	•154	518	089	•222	•435	259	-1.325	1.635	2.276	10
3	035	2.466	.223	237	879	531	389	544	1.363	1.021	1.649	-2.389	9
4	•295	030	-1.254	-1.339	354	172	•365	-1.482	333	2.226	355	-1.413	8
5	711	209	-•192	668	188	•340	.885	.827	-1.018	675	009	1.704	7
6	• 486	2.837	1.298	•816	320	1.469	•111	•337	1.706	754	2.076	1.908	6
7	1.427	1.085	<del>-</del> .572	.283	1.623	.878	•297	-3.141	•039	. 454	743	-1.649	5
8	657	645	-1.337	-1.146	•150	•067	480	•651	-2.510	•917	-3.722	•723	4
9	837	399	•677	•185	-1.096	459	324	178	-,494	•772	•923	•083	3
10	1.082	•230	• 284	• 786	003	126	397	-1.020	-1.179	• 265	4.527	<b></b> 768	2
11	•106	806	-1.664	•179	•504	.167	.735	1.403	555	.013	•649	463	1
12	803	• 346	•107	•976	•223	•156	003	•536	.800	•743	041	266	
n	1	2	3	4	5	6	7.	8	9	10	11	12	

TABLE III.- Continued

(f) December 1975

	12	11	10	9	8	7	6	5	4	3	2	1	m n
	1.034	526	.686	670	•254	•355	•207	398	•196	1.088	•243	021	12
1	1.814	.239	601	262	•579	•284	019	204	646	• 435	511	2.086	11
2	•495	1.016	1.493	416	•249	056	110	•544	•036	-1.310	•522	2.055	10
3	111	3.146	.077	533	•119	357	641	-1.084	1.936	308	.844	-2.688	9
4	1.394	•022	-2.405	2.759	•562	696	.038	719	-1.076	2.366	784	560	8
5	826	•298	230	-1.460	329	.581	•256	1.878	-2.098	904	418	2.587	7
6	• 3 76	3.381	2.797	969	•463	.245	•193	•691	2.058	-3.296	3.124	2.391	6
7	• 480	.933	064	•428	•5 <b>7</b> 9	677	•249	-2.359	012	•755	.102	-2.055	5
8	-1.398	-1.854	-1.940	-1.209	434	•109	056	1.703	-3.979	2.227	-3.666	32,1	4
9	457	-1.022	.187	•341	934	376	570	•469	•390	312	•306	1.042	3
10	1.214	•668	333	1.535	•177	•099	•073	-1.145	546	840	4.051	•826	2
11	•294	.070	695	•579	.668	•422	1.215	.334	207	•520	760	-2.102	1
12	619	210	1.127	160	•002	060	191	•604	•410	930	•163	•906	
n	1	2	3	4	5	6	7	8	9	10	11	12	

TABLE III.- Continued

(g) January 1976

	12	11	10	9	8	. 7	6	5	4	3	2	1	m/n
	.410	-1.128	315	.841	•162	201	1.251	470	•247	1.151	•222	.330	12
1	1.154	•503	669	• 971	•236	.010	473	.872	431	•908	•103	1.231	11
2	• 3 41	•653	036	408	413	•530	-1.014	1.003	•098	-1.087	•327	2.179	10
3	•161	2.558	.310	492	659	.341	066	-1.183	2.501	430	•514	-1.653	9
4	1.045	.837	-3.478	2.506	005	161	•593	-1.500	•167	1.914	932	-1.911	8
5	-1.991	•417	• 742	854	•503	060	199	•924	-3.438	827	248	1.336	7
6	397	2.080	3.359	•152	457	298	-1.600	1.975	•786	-3.111	3.057	2.763	6
. 7	1.865	•077	.077	.370	.861	•930	1.982	-2.145	1.703	•649	.826	-1.831	5
8	428	-1.358	-2.256	-1.263	•043	•343	•630	•751	-3.664	1.580	-2.037	409	4
. 9	995	218	-1.785	•431	-1.275	-1.023	-1.614	• 439	771	574	•053	•751	3
10	1.064	1.121	•012	1.433	•094	.609	-1.183	050	819	•075	2.319	•723	2
11	.611	376	1.203	279	•560	1.012	.221	.104	•120	218	.745	-1.027	1
12	319	034	•614	<b></b> 286	130	407	•562	-1.108	1.159	.032	484	-1.299	
n	1	2	3	4	5	6	7	8	9	10	11	12	

TABLE III.- Continued

(h) February 1976

///	12	11	10	9	8	7	6	5	4	3	2	1	n
	.210	-1.183	.197	160	•068	•402	. 417	828	.728	1.245	-1.060	518	12
1	1.869	.840	576	090	•145	1.030	797	.317	522	1.082	538	.489	11
2	.690	-2.107	•692	084	906	•357	.389	.988	448	-1.368	1.811	2.314	10
3	•319	2.229	1.780	• 277	982	169	628	498	1.846	471	.575	-1.376	9
4	.479	2.923	-2.418	2.732	512	•435	165	-1.200	.761	2.458	-1.759	-1.082	8
5	978	•533	.440	-1.105	-1.238	782	•963	1.431	-1.062	939	417	.296	7
6	.582	1.512	4.196	944	334	718	-1.555	1.243	1.201	-2.612	4.009	2.647	6
7	1.748	•094	395	1.284	1.363	148	• 390	-3.316	.152	1.137	1.892	130	5
8	.359	-1.253	-2.143	524	210	228	074	.010	-2.217	1.543	-3.014	.198	4
9	-1.619	.409	021	139	-1.510	298	416	. 166	.756	-1.195	374	754	3
10	1.116	1.237	552	1.399	.042	•102	648	.205	442	•919	2.976	470	2
11	• 094	-1.104	168	.373	.761	034	.034	.438	062	-1.382	1.287	969	1
12	792	552	.270	997	032	626	.249	.725	.340	378	212	-1.808	
n m	1	2	3	4	5	6	7	8	9	10	11	12	

TABLE III.- Continued

# (i) March 1976

7///		<del></del>	<del></del>				·						m
	12	11	10	9	8	7	6	5	4	3	2	1	n
	1.258	231	189	.621	•326	. 768	•761	276	243	.174	-1.539	•263	12
1	2.067	. 8 59	.857	•264	627	•565	511	•354	496	.492	.872	-1.153	11
2	1.122	-1.791	. 421	.760	292	162	.036	.337	.383	-1.160	2.918	2.213	10
3	146	.951	.760	•977	•595	-1.027	051	-1.013	.825	.078	.170	-1.136	9
4	• 549	2.554	-1.444	1.783	•157	763	.385	-1.638	-1.652	1.851	-2.549	-1.148	8
5 n	388	1.298	1.018	654	172	2.418	•547	2.115	934	-1.295	• 077	-1.241	7
6	•109	438	2.355	336	522	1.078	-1.089	1.513	2.201	-2.184	4.603	3.380	6
7	.214	-1.462	-1.094	• 275	2.061	011	804	-3.280	.041	1.466	1.071	•932	5
8	398	.204	369	200	498	197	131	.717	-3.223	. 223	-3.054	1.096	4
9	.071	1.879	1.192	•221	347	273	•560	• 199	1.122	-1.324	.025	-1.318	3
10	1.474	.630	430	1.007	.493	. 364	074	779	.633	.411	2.261	481	2
11	327	846	391	. 690	051	.188	055	.700	•579	757	.029	.332	1
12	092	357	.894	114	•316	194	169	506	1.066	.170	384	.753	
n	1	2	3	4	5	6	7	8	9	10	11	12	

# TABLE III.- Continued

# (j) April 1976

	12	11	10	9	8	7	6	5	4	3	2	1	m
	.026	-1.477	357	451	.154	461	445	186	.027	.416	-1.300	1.216	12
1	2.628	349	406	506	213	•719	•703	.351	354	.383	682	•176	11
2	1.542	-1.455	•307	•437	.217	340	•342	.491	363	-1.162	2.096	1.348	10
3	.094	815	1.133	.148	387	-1.027	542	876	.244	.172	1.111	-1.763	9
4	.227	1.432	-1.713	.273	492	283	•151	933	451	1.673	-2.063	653	8
5	684	.057	549	•269	-1.244	1.656	•257	1.071	367	-1.225	549	577	7
6	• 220	799	3.118	158	•149	.861	544	.053	1.118	-1.541	3.175	2.997	. 6
7	.501	775	.070	•051	2.185	702	723	-3.366	.290	2.037	.731	.179	, 5
8	-1.087	•456	837	.031	. 459	156	019	1.285	-2.357	.757	-1.029	1.301	4
9	•005	•960	•642	335	894	•474	.012	020	• 058	934	1.630	-1.502	3
10	1.389	•659	107	.118	208	510	293	941	774	-1.913	.950	.077	2
11	244	-1.211	223	.241	•250	279	.361	•579	047	059	-1.371	.200	1
12	154	939	•536	. 476	016	•179	.207	197	.347	1.582	722	341	
n/m	1	2	3	4	5	6	7	8	9	10	11	12	

TABLE III.- Continued

(k) May 1976

	12	11	10	9	8	7	6	5	4	3	2	1	m
	.397	.073	. 958	230	386	089	• 4 42	.008	798	.174	.016	.718	12
1	3.111	.616	.030	.652	.486	.311	.047	660	381	582	•006	128	11
2	2.977	. 420	•078	700	•490	.021	208	• 3 46	.013	361	.214	1.824	10
3	199	266	.462	.385	591	210	061	308	.681	1.162	.721	567	9
4	•045	420	-2.444	304	838	165	.220	367	538	1.061	373	202	8
n 5	-1.187	-1.494	.482	.104	.827	074	.989	•987	404	-1.196	830	705	7
n 6	•062	•185	3.010	•636	•391	.415	•796	355	1.153	250	1.277	1.689	6
7	•417	1.132	319	.316	.342	. 319	742	-1.304	-1.389	•931	.702	-1.943	5
8	415	1.578	-1.827	172	360	1.154	.151	255	-2.768	097	890	1.729	4
9	927	• 50 6	.240	.045	-1.162	.371	310	-1.534	344	278	1.254	572	3
10	.368	.631	•414	•352	136	967	758	882	579	•166	3.455	.285	2
11	.846	-1.434	360	.317	•329	.293	.615	• 592	.817	433	.166	.981	1
12	• 432	931	•553	.089	•52 <b>7</b>	•5 <b>7</b> 9	417	1.351	.277	. 279	217	.249	
n	1	2	3	4	5	6	7	8	9	10	11	12	-///

TABLE III.- Concluded

(1) June 1976

	12	11	10	9	8	7	6	5	4	3	2	1	m/n
	481	585	-1.568	618	508	507	240	.393	-1.388	.439	442	• 400	12
1	4.919	.332	117	039	957	413	425	018	.014	323	.213	314	11
2	4.066	2.849	1.567	-1.614	•063	•762	267	183	1.253	.268	1.568	1.732	10
3	3 -1.161	2.011	• 364	408	•485	.158	•149	333	1.396	•698	1.084	950	9
, 4	987	-1.707	-1.025	-3.683	505	•367	•460	-1.053	•196	589	•091	-1.388	8
5	-2.019	-3.384	934	-1.472	687	087	•328	100	228	-1.754	189	255	7
6	-1.348	525	1.751	•984	165	.885	.283	.812	-1.249	104	958	2.084	6
7	.745	•948	1.301	1.403	.657	.412	196	423	-2.077	2.313	-2.675	-2.773	5
8	• 468	2.182	-1.241	•968	.061	1.466	•292	520	-1.192	.251	.200	1.495	4
9	.744	1.125	586	.041	005	•274	•530	432	1.199	389	5.304	1.741	3
10	.840	586	• 941	442	•557	070	1.051	954	360	424	5.778	1.262	2
11	511	-1.621	159	.004	.413	•099	•364	-1.250	974	.176	765	512	1
12	.196	•486	.313	•979	029	• 466	652	483	•374	1.011	-1.429	322	
n	1 -	2	3	4	5	6	7	8	9	10	11	12	

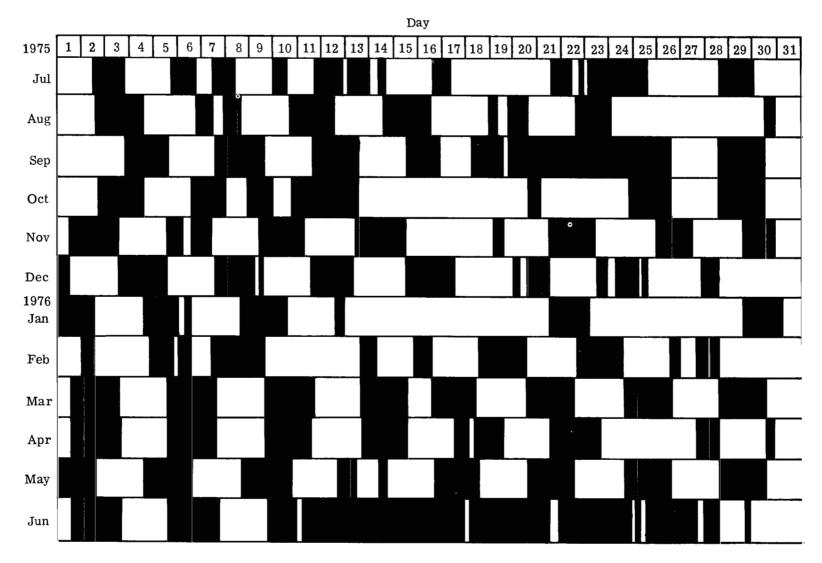


Figure 1.- Time periods for longwave radiation measurements from Nimbus 6 (dark bands) for 12-month period from July 1975 through June 1976.

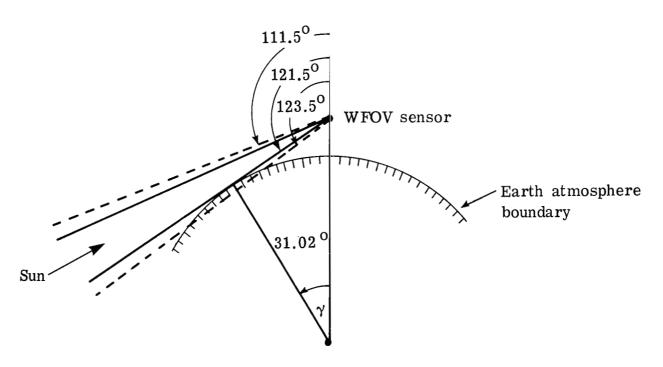


Figure 2.- Range of Sun zenith angle for which wide-field-of-view (WFOV) sensor measurements were deleted (111.5 $^{\circ}$  to 123.5 $^{\circ}$ ).

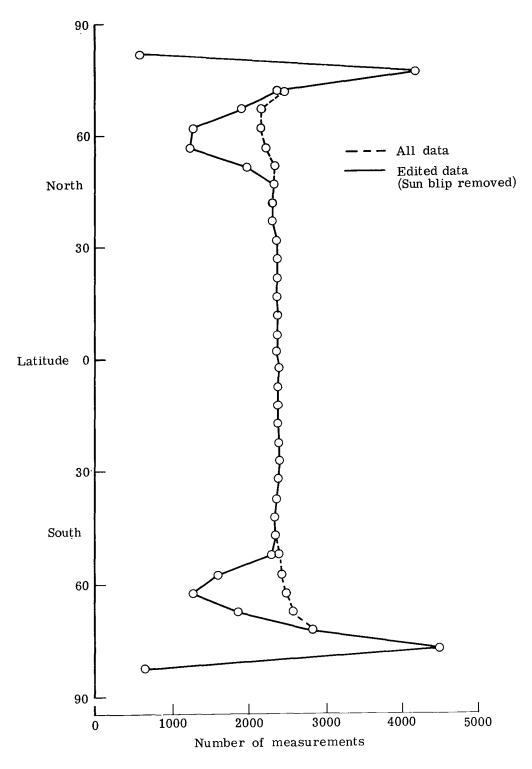


Figure 3.- Zonal sampling plot for September, showing zonal deficiency in number of measurements due to removal of Sun contaminated measurements.

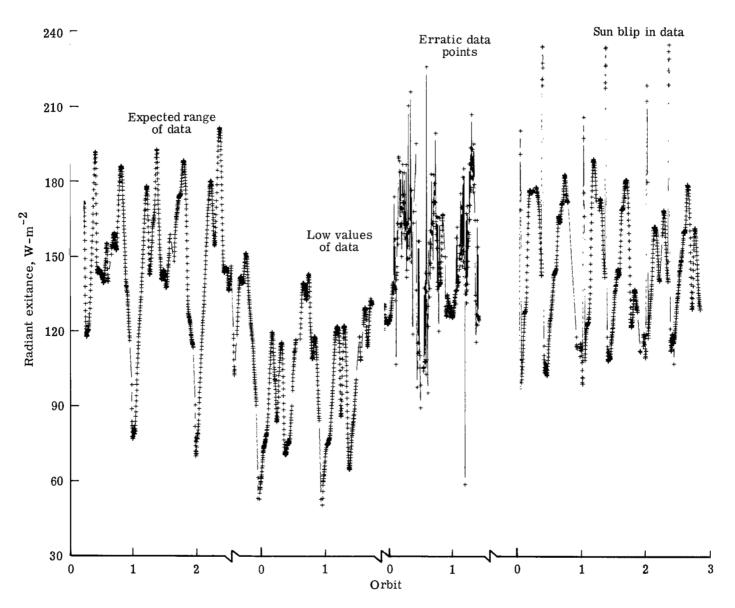


Figure 4.- Composite plot of raw longwave radiant exitance showing some anomalies in data.

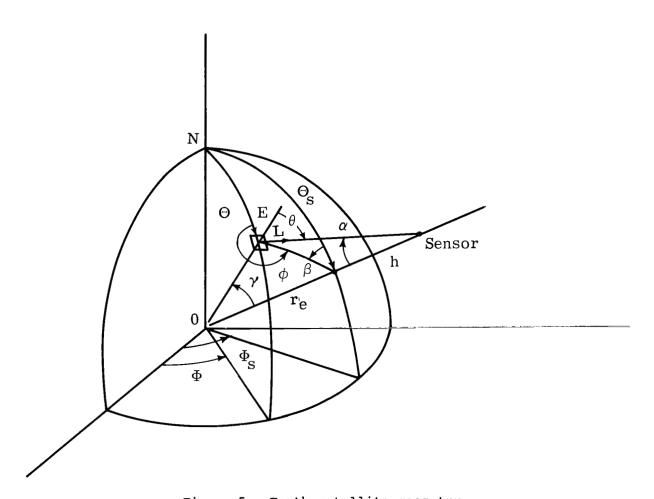


Figure 5.- Earth satellite geometry.

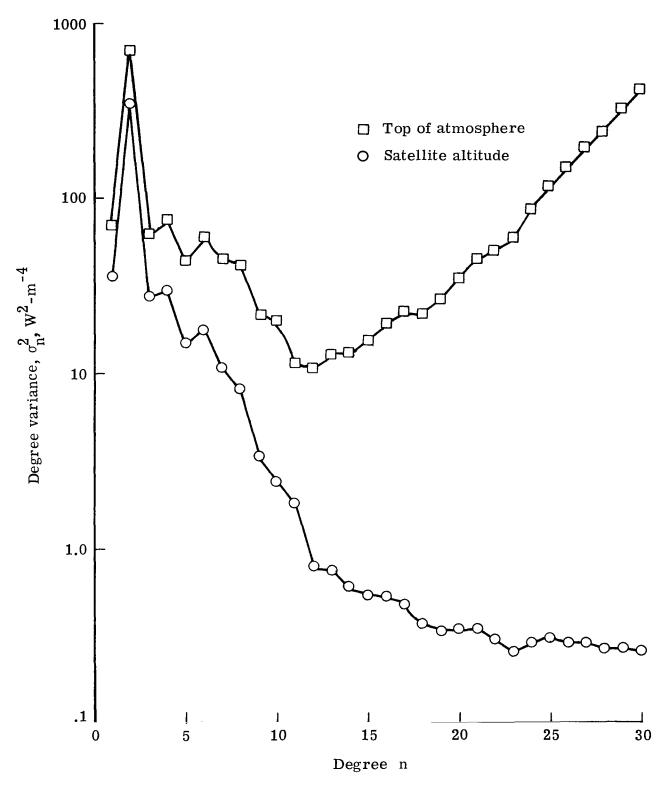
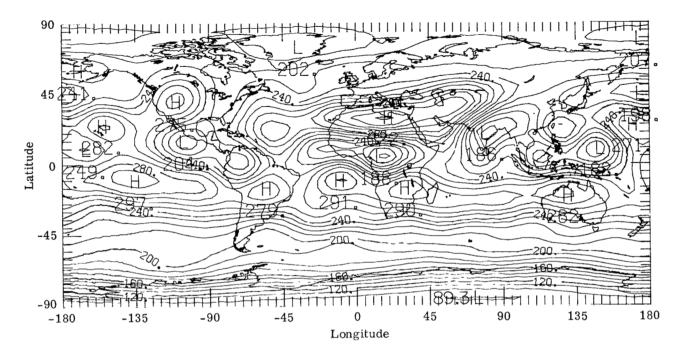


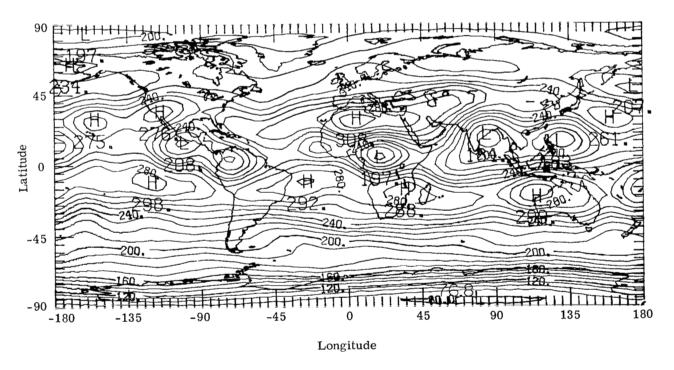
Figure 6.- Degree variance plot for average of 12-month data set.

1 11 11 11 11 11 11 11 11 1

Figure 7.- Standard deviation of average degree variance over 12 months at top of atmosphere for degree 1 to 30.

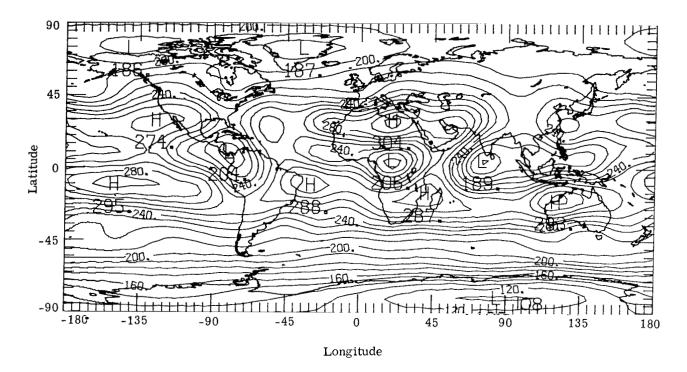


(a) July 1975.

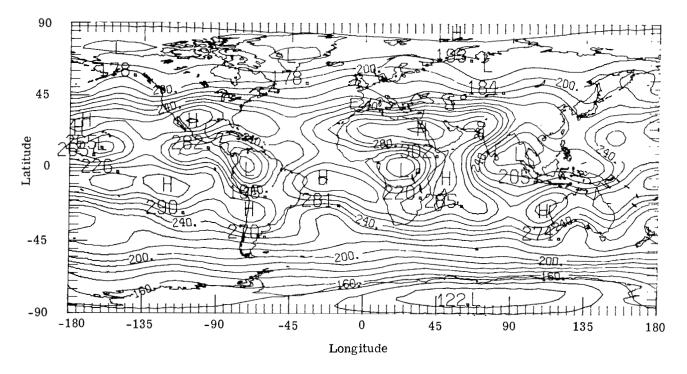


(b) August 1975.

Figure 8.- Geographical distribution of Earth emitted radiant exitance at top of atmosphere (W-m $^{-2}$ ) for 12-month period.



# (c) September 1975.

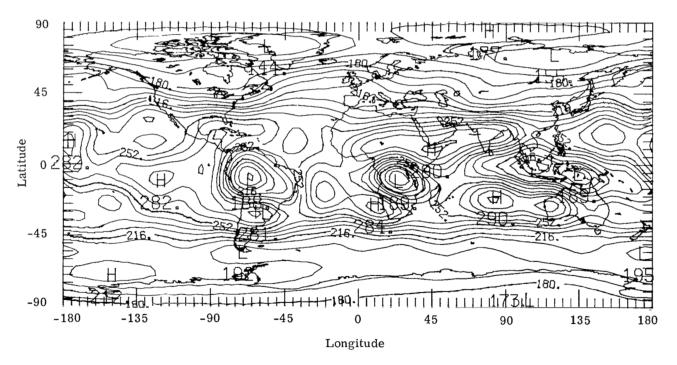


(d) October 1975.

Figure 8.- Continued.

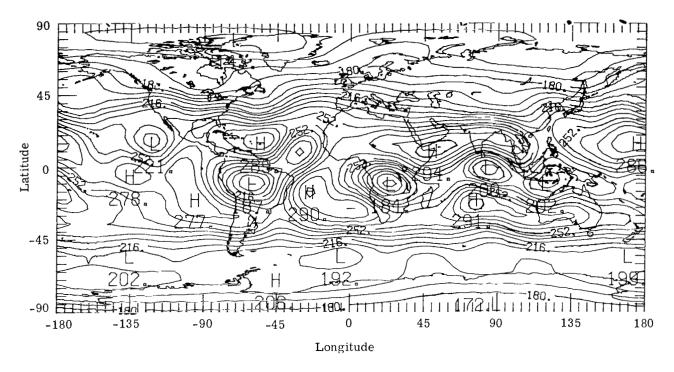
45 Latitude 0 224 -45 192. -135 -90 -180 -45 45 90 135 180 Longitude

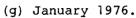
#### (e) November 1975.

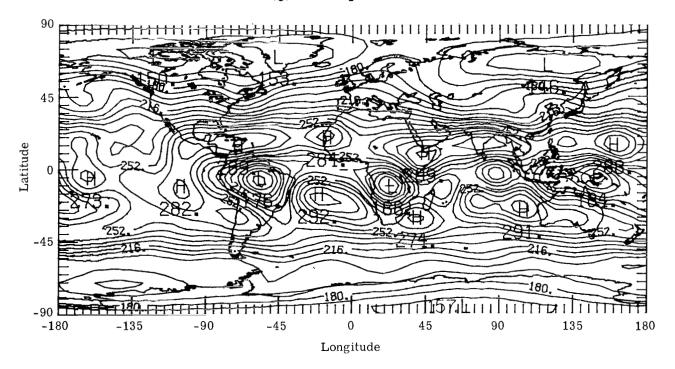


(f) December 1975.

Figure 8.- Continued.

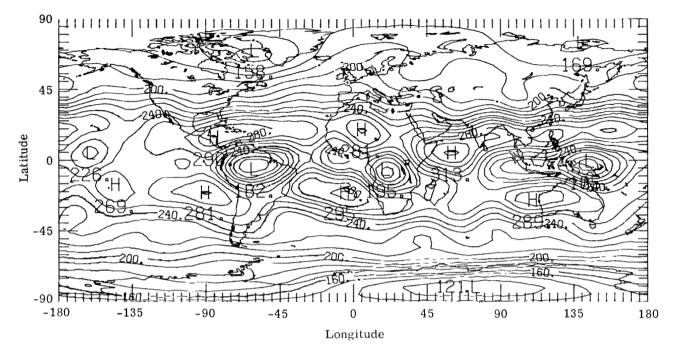




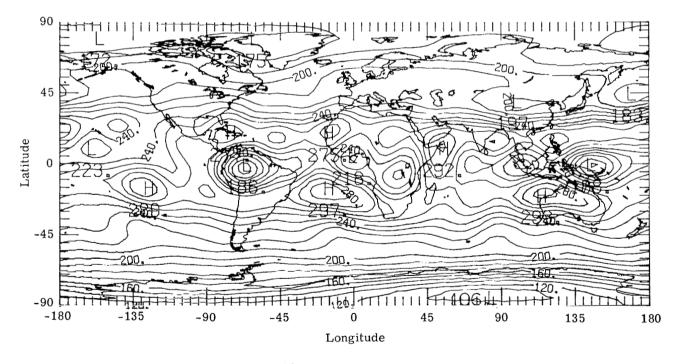


(h) February 1976.

Figure 8.- Continued.



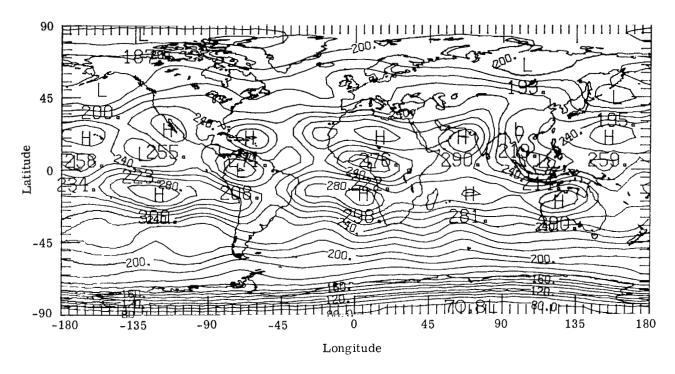
(i) March 1976.



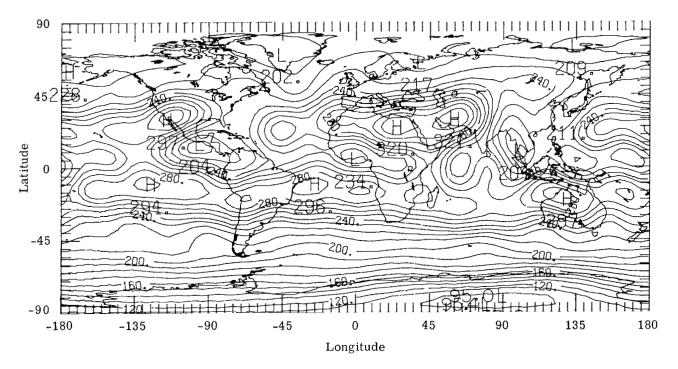
(j) April 1976.

Figure 8.- Continued.





(k) May 1976.



(1) June 1976.

Figure 8.- Concluded.

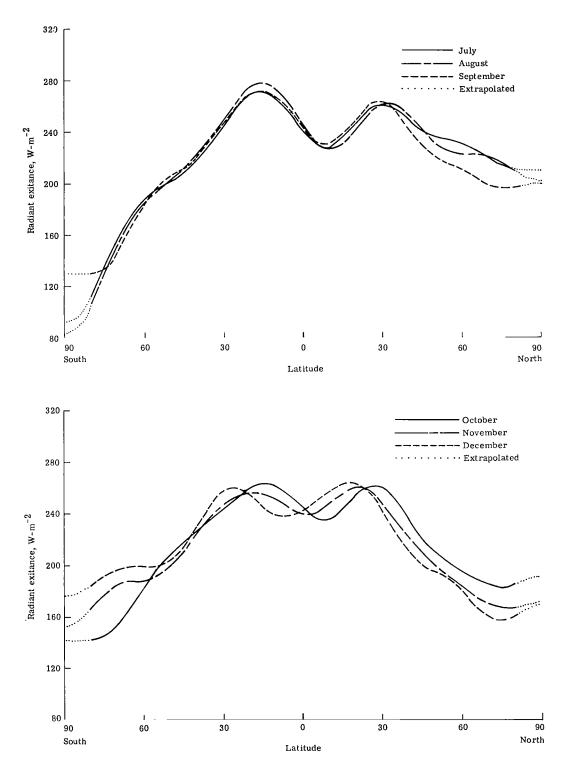
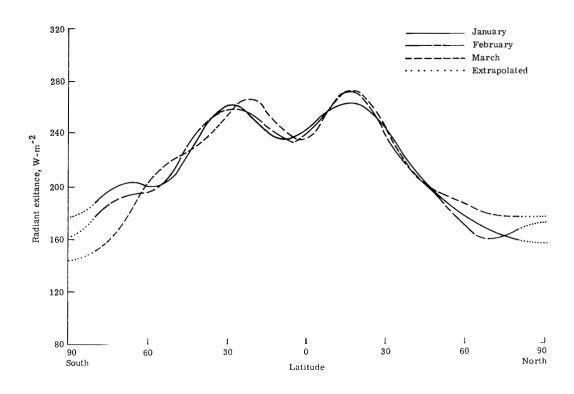


Figure 9.- Monthly averages of zonal emitted radiant exitance  $(W-m^{-2})$ .



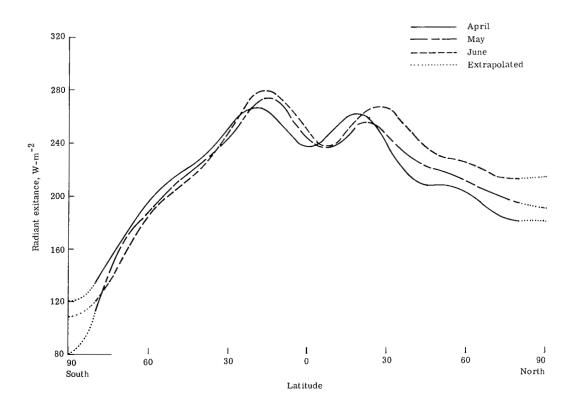


Figure 9.- Concluded.

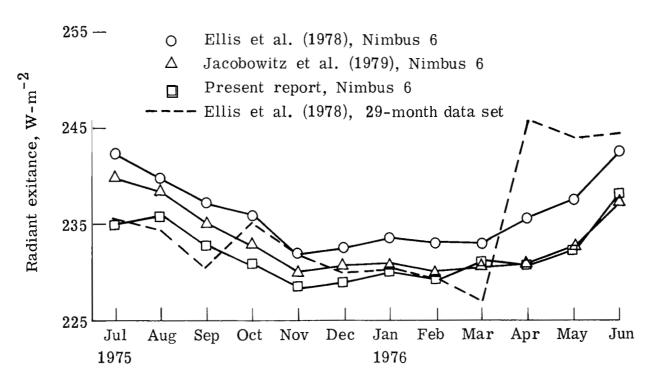


Figure 10.- Time histories of global mean from Nimbus 6 wide-angle radiometer for July 1975 through June 1976 (29-month composite data set for 1964 through 1971 represented by dashed line).

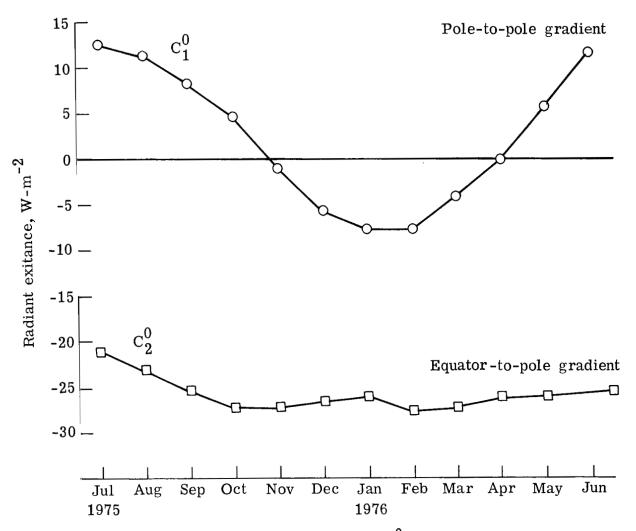


Figure 11.- Time histories of pole-to-pole gradient  $c_1^0$  and equator-to-pole gradient  $c_2^0$  for July 1975 through June 1976.

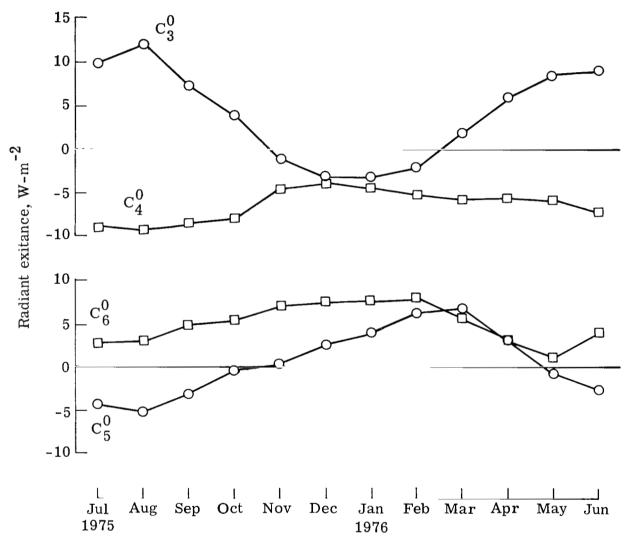


Figure 12.- Time histories of  $\ C_n^0$  for n=3 to n=12 for July 1975 through June 1976.

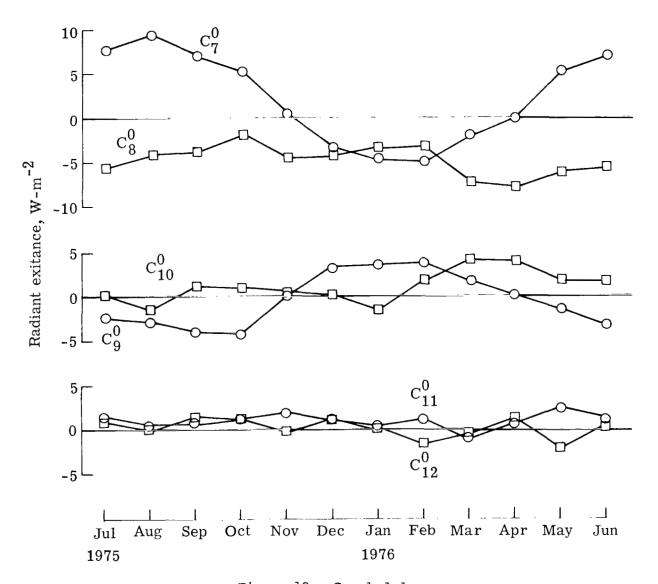


Figure 12.- Concluded.

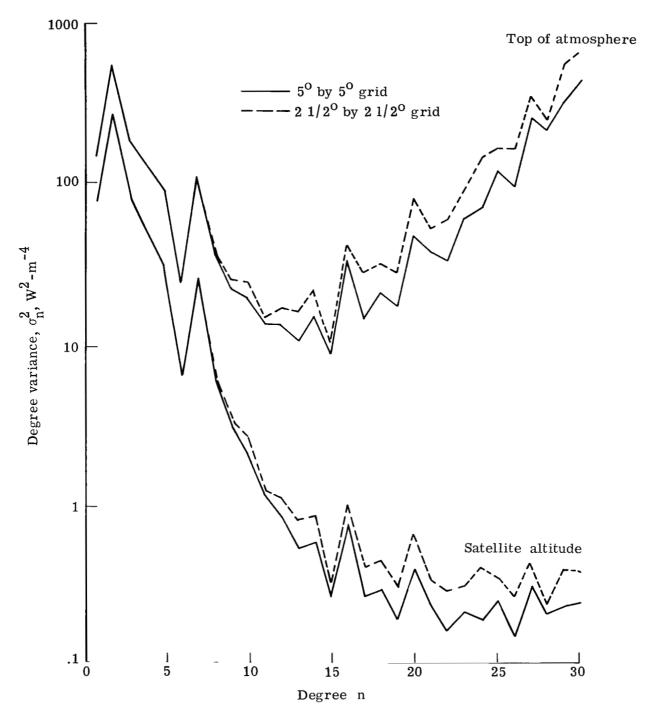


Figure 13.- Comparison of degree variance plots for August 1975 for  $$1^{\rm O}$$  and 2- grid systems.

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